EFFECT OF GRAPHICA3D ON STUDENTS' VISUALIZATION AND SKETCHING OF 3D GRAPHS IN MULTIVARIABLE CALCULUS

*Siti Mariam Saad[1], Siti Nurleena Abu Mansor[2] and Ahmad Rashidi Azudin[3]
**smariam.saad@uitm.edu.my[1], sitin140@uitm.edu.my[2], ahmadrashidi@uitm.edu.my[3]*

[1,2,3]Jabatan Sains Komputer & Matematik (JSKM),
Universiti Teknologi MARA Cawangan Pulau Pinang, Malaysia

**Corresponding author*

ABSTRACT

Developing skills in visualizing, interpreting, and sketching three-dimensional (3D) graphs is essential for learning multiple integrals, a key topic in multivariable calculus. However, many students struggle with these skills, creating barriers to understanding and solving related problems. While dynamic mathematics software like GeoGebra can support visualization, existing teaching materials often lack structured integration of technology to strengthen 3D graph learning. To address this gap, an interactive module, Graphica3D, was developed. The module combines explanatory notes on 3D graphs with embedded GeoGebra applets that enable interactive visualization. This study evaluated the effectiveness of the Graphica3D module using a pre–post test design. Participants were 44 engineering students enrolled in Further Calculus for Engineers at UiTM Cawangan Pulau Pinang. Data were analyzed using a non-parametric test, namely ANOVA-type statistics and results showed significant improvements in students' ability to visualize and sketch 3D graphs. Moreover, no interaction between test performance (pre vs. post) and gender was observed, suggesting that the module benefits students equitably. These findings demonstrate that Graphica3D is a valuable supplementary tool for enhancing spatial visualization in multivariable calculus and for supporting inclusive teaching practices in higher education mathematics.

Keywords: 3-dimensional graphs, GeoGebra, interactive learning, multiple integrals, multivariable calculus

Introduction

Multivariable calculus is a compulsory subject in science and engineering degree programs, and one of its key topics is multiple integrals. These integrals involve integrating functions of more than two variables over two-dimensional (2D) or three-dimensional (3D) regions. Before evaluating a multiple integral, students must first understand the region of integration to determine the limits of integration, which requires accurate interpretation of 3D graphs (Milenkovic et al., 2023).

However, many students face difficulties in developing the skills to visualize, interpret, and sketch 3D graphs represented by algebraic equations. They often struggle to connect algebraic equations with their corresponding geometric representations (Gemechu et al., 2021). This difficulty has been identified as a significant barrier to learning and problem-solving in multiple integrals (Kashefi et al., 2010; Listiana et al., 2022), particularly among students from non-mathematics programs (Milenkovic et al., 2023).

To address this issue, dynamic mathematics software has been increasingly adopted to support visualization in calculus learning. Software such as MATLAB, Mathematica, Maple, and GeoGebra provide interactive environments that allow students to explore mathematical objects independently (Milenković & Vučićević, 2024). Among these, GeoGebra has gained wide adoption in mathematics education due to its open-source nature and versatile applications in geometry, algebra, and calculus (Hohenwarter et al., 2008; Ziatdinov & Valles Jr., 2022). GeoGebra can also be seamlessly integrated into teaching materials.

Building on this potential, we developed Graphica3D, an interactive module with embedded GeoGebra applets designed to help students visualize, interpret, and sketch 3D graphs. Initial survey results from engineering students enrolled in multivariable calculus course at UiTM Cawangan Pulau Pinang (UiTMCPP) indicated that Graphica3D was perceived as an effective supplementary learning tool that enhanced their visualization and sketching skills (Saad et al., 2025, 2024).

While these findings highlight students' positive perceptions, further investigation is needed to determine the module's impact on learning outcomes. Therefore, the present study aims to evaluate students' improvement in sketching 3D graphs after using the Graphica3D module. The module was implemented in the Further Calculus for Engineers course, and students' sketching performance was assessed before and after its use. Specifically, this study examines whether there is a statistically significant difference between pre-test and post-test scores, as well as whether gender differences influence students' performance.

The research questions guiding this study are:

1. Do students' skills in sketching 3D graphs improve from pre-test to post-test?
2. Do gender differences influence students' scores in sketching 3D graphs?
3. Is there an interaction between test performance (pre-test and post-test) and gender?

Methodology

The Graphica3D module, developed as a digital flipbook, was designed to address students' challenges in visualizing and understanding 3D graphs. It incorporates interactive 3D visualizations supported by GeoGebra applets, along with explanatory notes on common 3D surfaces such as planes, cones, cylinders, paraboloids, spheres, and hemispheres. Figure 1 presents sample pages from the Graphica3D module.

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Figure 1: Graphica3D module

A pre–post study design was employed to measure students' improvement in sketching 3D graphs after using the Graphica3D module. Students' sketches were collected both before and after using the module. The same set of six algebraic equations (Table 1) was used in the pre-test and post-test, and students were instructed to sketch the corresponding 3D graphs.

Table 1: Algebraic Equations for Pre- and Post-Tests

No	Algebraic Equations
1	$y = x + 1, x = -1$
2	$y = 2 - \sqrt{x^2 + z^2}$
3	$x = y^2 + z^2 - 3$
4	$x^2 + z^2 = 16$
5	$y = -\sqrt{4 - x^2 - z^2}$
6	$z = -\sqrt{4 - x^2 - y^2}$

A total of 44 students participated in the study, comprising 23 females and 21 males from three engineering programs at UiTMCPP: Civil Engineering, Chemical Engineering, and Electrical Engineering. For the pre-test, students were asked to sketch 3D graphs of six algebraic equations (Table 1) prior to the introduction of the module. After completing the pre-test, students were introduced to the

Graphica3D module and instructed to review it. One week later, they completed the post-test by sketching graphs of the same equations.

Each sketch was evaluated using a scoring rubric with three criteria: (i) axes orientation, (ii) basic shape accuracy, and (iii) correct representation of mathematical features such as symmetry, intercepts, and curves. For each student, total scores were computed for both pre-test and post-test. Descriptive statistics (mean and standard deviation) were calculated for both sets of scores.

To address the research questions, a non-parametric test, the ANOVA-type statistic (ATS) (Brunner & Puri, 2001), was used as an alternative to mixed ANOVA (two-way ANOVA with repeated measures), given that the data did not meet the normality assumption. The ATS was appropriate for this study because it allows examination of interactions between gender (male, female) and test at two different times (pre-test, post-test). Data analysis was conducted using the nparLD package in R (Noguchi et al., 2012).

Results

The effectiveness of the Graphica3D module in enhancing students' ability to visualize and sketch 3D graphs was evaluated using a pre–post study design. Students' scores were analyzed with the non-parametric ANOVA-type statistic (ATS) to examine the effects of test (pre vs. post), gender, and their interaction.

Table 2 presents the means and standard deviations (SD) of test scores. The overall mean pre-test score was 14.48 (SD = 8.40), while the post-test mean increased to 26.18 (SD = 8.85). For female students, the mean pre-test score was 13.30 (SD = 8.62) and the post-test mean was 24.48 (SD = 10.55). Male students scored slightly higher, with a pre-test mean of 15.76 (SD = 8.17) and a post-test mean of 28.05 (SD = 6.25).

Table 2: Descriptive Statistics of Pre- and Post-Test Scores by Gender

Test	Gender	Mean	Std. Deviation	n
Pre	Female	13.30	8.62	23
	Male	15.76	8.17	21
	Total	14.48	8.40	44
Post	Female	24.48	10.55	23
	Male	28.05	6.25	21
	Total	26.18	8.85	44

Results of the ATS are shown in Table 3. A significant main effect was found for test, F = 144.49, p-value < 10^{-33}, indicating that post-test scores were significantly higher than pre-test scores at the 0.05 significance level. Gender had no significant effect on test scores, F = 1.40, p-value = 0.237, suggesting no overall performance difference between male and female students. The interaction between test and gender was also not significant, F = 0.40, p-value = 0.527, indicating that the improvement from pre-test to post-test was consistent across genders. Figure 2 provides a graphical illustration of the interaction effect. The parallel lines confirm the absence of a significant interaction between test and gender on students' test performance.

Table 3: ATS Results for Test Occasion (Pre vs Post), Gender, and Interaction Effects

Source	df	F	p-value
Test	1	144.4877	$<10^{-33}$
Gender	1	1.3983	0.2370
Test * Gender	1	0.3999	0.5271

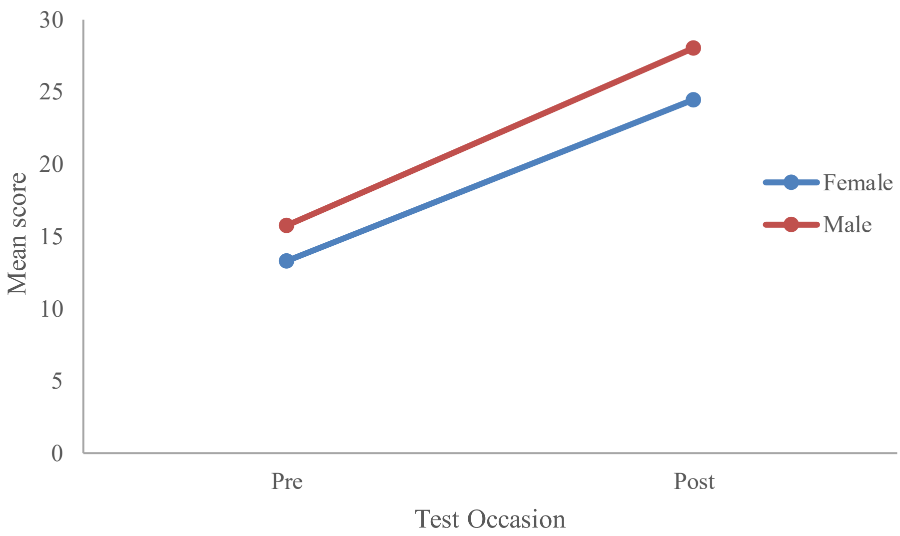

Figure 2: Interaction Effect of Pre- and Post-Test Scores by Gender

Discussion

This study examined whether interactive teaching materials, specifically the Graphica3D module, could help students improve their spatial skills in visualizing, interpreting, and sketching 3D graphs. The descriptive statistics revealed an overall mean gain of 11.71 points from pre-test to post-test. This improvement was further confirmed by the non-parametric ATS test, which showed a statistically significant difference between pre-test and post-test performance. These results indicate that the

Graphica3D module effectively enhances students' visualization and sketching skills, which are essential for understanding multiple integrals. These findings are consistent with previous research showing that dynamic mathematics software such as GeoGebra supports deeper learning of multiple integrals compared to traditional methods (Listiana et al., 2022; Milenković et al., 2024; Septian et al., 2021).

With regard to gender differences, descriptive statistics indicated that male students scored slightly higher than female students on both tests, with mean gain of 12.29 and 11.18, respectively. However, the ATS test revealed no statistically significant gender difference. This suggests that both male and female students developed comparable visualization and sketching skills. Previous studies have reported mixed findings in this area. For example, Gamage and Charles-Ogan (2019) and Akpan et al. (2022) concluded that gender does not influence mathematics performance when using GeoGebra, emphasizing that male and female students possess similar cognitive abilities to learn geometry effectively. In contrast, Adelabu et al. (2019) reported that female students outperformed male students in mathematics when GeoGebra was integrated into instruction. Research directly comparing male and female performance in multivariable calculus learning with dynamic mathematics software remains limited.

Lastly, the investigation of interaction effects between pre-test and post-test and gender revealed no significant interaction. This means that both male and female students benefited similarly from the Graphica3D module. Overall, these results complement prior research on GeoGebra-assisted learning and extend the evidence by demonstrating that the Graphica3D module is equally effective for both male and female students.

Conclusion

This study evaluated the effectiveness of the Graphica3D module as a supplementary learning tool in multiple integrals, a key topic in multivariable calculus. Results showed that students demonstrated significant improvements in their ability to visualize and sketch 3D graphs after using the module.

These findings highlight the value of integrating technology-based modules such as Graphica3D into the teaching and learning of multivariable calculus. By enhancing students' ability to visualize and sketch 3D graphs, the module not only strengthens conceptual understanding but also helps overcome a key barrier to learning multiple integrals. Importantly, the results indicate that the Graphica3D module provides equitable learning benefits across genders, supporting inclusive teaching practices in higher education mathematics.

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